A brief summary of the main aspects of the long range voter model and the p−voter model for Workshop SMOD 2026

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Abstract

Statistical mechanics was born to provide a powerful theoretical framework to describe equilibrium collective phenomena emerging from the interaction of many degrees of freedom in thermodynamical or chemical systems. In non-equilibrium settings there is no comprehensive theory yet, but we can study each system individually by investigating its time evolution (dynamics) by means of stochastic equations that involve, in some way, the time evolution of the degrees of freedom of the system. We focus on ordering processes, one of the simplest yet most paradigmatic classes of non-equilibrium dynamics. These processes are relevant not only in physics, where they model the relaxation toward ordered equilibrium phases, but also in interdisciplinary contexts such as social and biological [5,6] dynamics.

We first study analytically the ordering kinetics of the long-range voter model in arbitrary dimensions [2,3,4]. The model consists of N agents (or spins) described by binary variables $S_i = \pm 1$, where at each time step an agent copies the state of another chosen at distance r with probability $P(r) \propto r^{-\alpha}$, with $\alpha > 0$. The long-range voter model is exactly solvable and exhibits nontrivial ordering behavior depending on the interaction exponent α.

Although similar in spirit to the Ising model − the physical model used to describe ordering in ferromagnets − the dynamics of voter model turns out to be different from the Ising one. We then introduce and investigate a generalization of the voter dynamics, the long-range p−voter model [1]. In this model, each agent adopts the state of the majority of p other agents selected at distances distributed according to the same algebraic probability $P(r)$. For $p = 2$, the dynamics can be exactly mapped onto the standard long-range voter model. For $p \geq 3$ the equations for correlation functions do not close and we employ numerical simulations to characterize the ordering kinetics. We find that the system crosses over to the dynamical universality class of the Ising model with long-range coupling $J(r) \propto r^{-\alpha}$.

Our results highlight the emergence of a transition in dynamical universality controlled by the parameter p, underscoring the role of simple, analytically tractable models as building blocks for understanding more complex non-equilibrium phenomena. Owing to universality, insights gained from these minimal models are expected to be relevant across a wide range of physical, biological, and social systems.

References

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